MODULAR FUNDAMENTAL DOMAIN

BERNDT E. SCHWERDTFEGER

ABSTRACT. Drawings for the modular fundamental domain with METAPOST

PREFACE

This note contains figures of the modular fundamental domain drawn with METAPOST.

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1. THE MODULAR FUNDAMENTAL DOMAIN

In this note I will depict the *modular fundamental domain* $\mathcal{D}$ of $\mathbf{SL}(2,\mathbf{Z})$ operating on the Poincaré upper half plane $\mathcal{H}$ drawn by METAPOST.

Recall that $\mathcal{D} = \{x \in \mathcal{H} \mid |\mathrm{Re}\, x| \le \frac{1}{2}, |x| \ge 1\}$ is the region delimited by the unit circle and the two straight lines thru $-\frac{1}{2}$ and $+\frac{1}{2}$, see e.g. GUNNING [1, I, §2], LAMOTKE [2, 5.1.3], SERRE [3, VII, §1.2].

$$\begin{aligned} \mathbf{SL}(2,\mathbf{Z}) \times \mathcal{H} &\longrightarrow \mathcal{H} \\ (\gamma, x) &\longmapsto \gamma \cdot x \end{aligned}$$

where

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot x = \frac{ax+b}{cx+d}$$

We note $S = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$ and $T = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$, i.e. $S \cdot x = -1/x$ and $T \cdot x = x+1$.

$$ST = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \qquad (ST)^2 = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \qquad (ST)^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$TS = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \qquad (TS)^2 = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} \qquad (TS)^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

1.1. Drawing with METAPOST.

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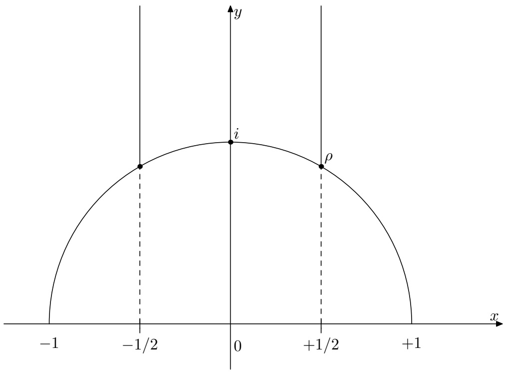
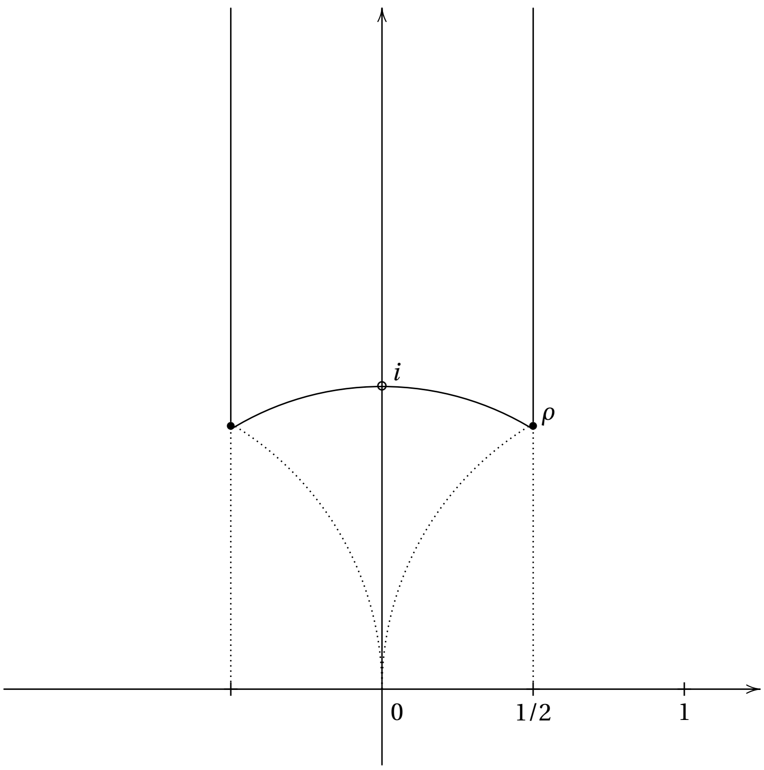
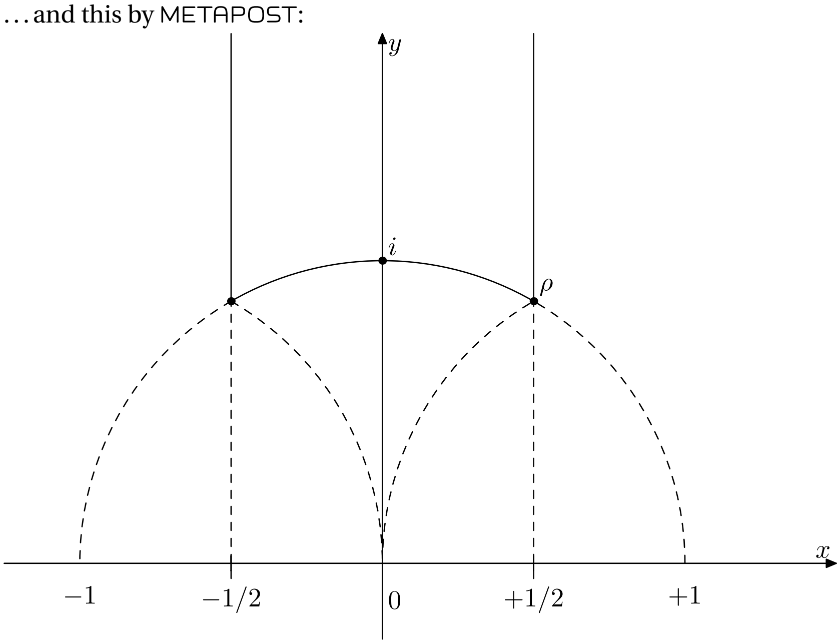


This is drawn with XY:



. . . and this by METAPOST:



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