MODULAR FUNDAMENTAL DOMAIN

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ABSTRACT. Drawings for the modular fundamental domain with METAPOST

PREFACE

This note contains figures of the modular fundamental domain drawn with METAPOST.

Berlin, May 2, 2011 B. E. Schwerdtfeger

1. THE MODULAR FUNDAMENTAL DOMAIN

In this note I will depict the *modular fundamental domain* $\mathcal{D}$ of $\mathbf{SL}(2,\mathbb{Z})$ operating on the Poincaré upper half plane $\mathbb{H}$ drawn by METAPOST.

Recall that $\mathcal{D} = \{x \in \mathbb{H} \mid |\operatorname{Re} x| \leq \frac{1}{2}, |x| \geq 1\}$ is the region delimited by the unit circle and the two straight lines thru $-\frac{1}{2}$ and $+\frac{1}{2}$, see e.g. GUNNING [1, I, §2], LAMOTKE [2, 5.1.3], Serre [3, VII, §1.2].

$$\begin{aligned} \mathbf{SL}(2,\mathbb{Z}) \times \mathbb{H} &\longrightarrow \mathbb{H} \\ (\gamma, x) &\longmapsto \gamma \cdot x \end{aligned}$$

where

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot x = \frac{ax+b}{cx+d}$$

We note $S = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$ and $T = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$, i.e. $S \cdot x = -1/x$ and $T \cdot x = x + 1$.

$$\begin{aligned} ST &= \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} & (ST)^2 &= \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} & (ST)^3 &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \\ TS &= \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} & (TS)^2 &= \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} & (TS)^3 &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

1.1. Drawing with METAPOST.

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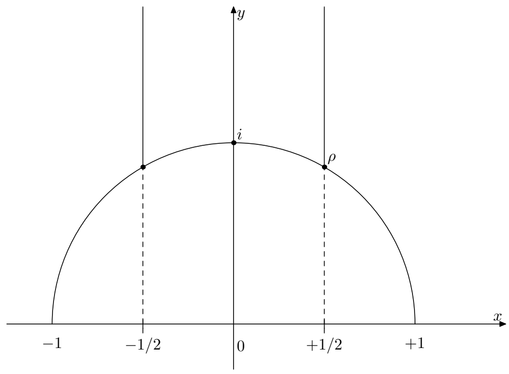
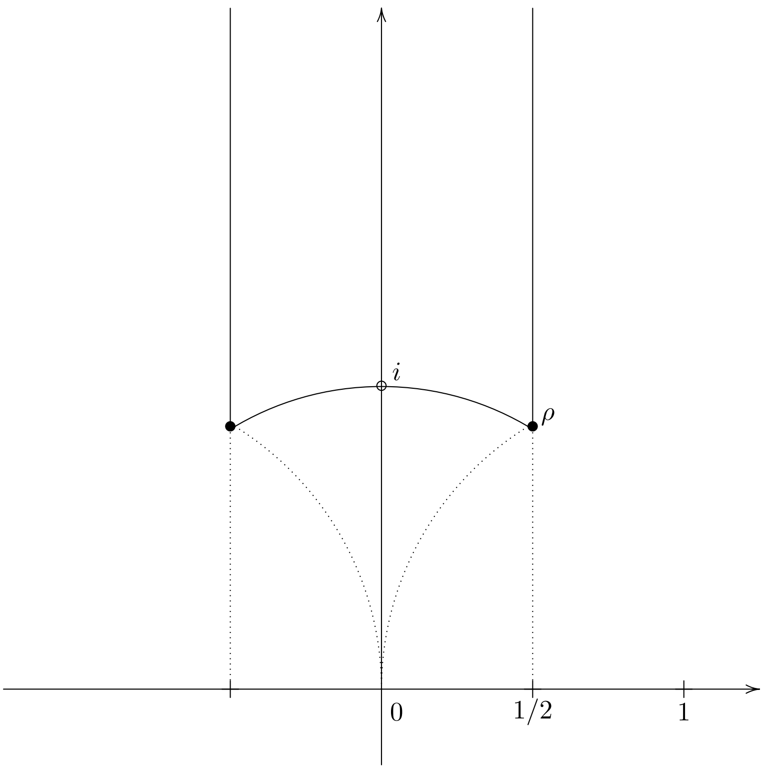
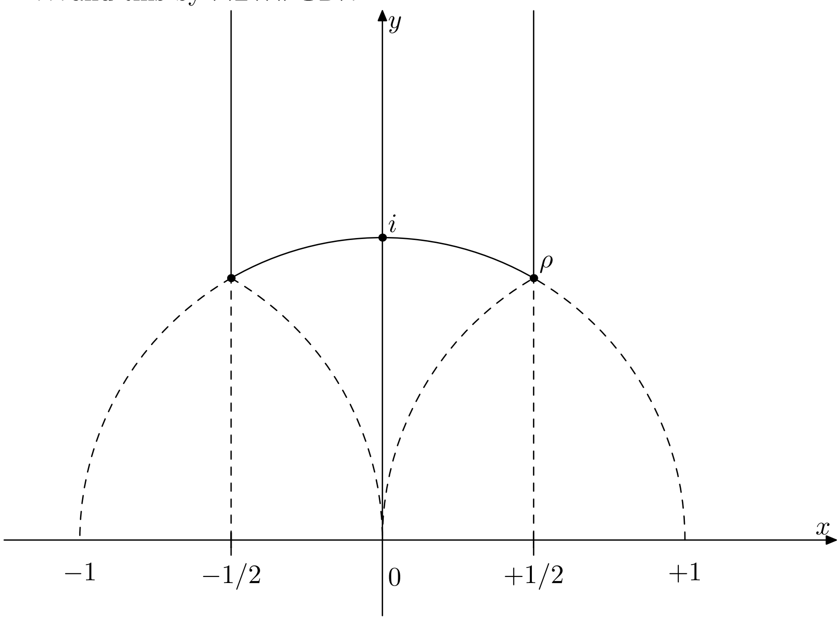


This is drawn with XY:



...and this by METAPOST:



References

[1] Robert C. Gunning, *Lectures on Modular Forms*, Annals of Mathematics Studies, vol. 48, Princeton University Press, Princeton, 1962.

[2] Klaus Lamotke, *Riemannsche Flächen*, Springer, 2005.

[3] Jean-Pierre Serre, *Cours d'arithmétique*, Presses Universitaires de France, Paris, 1970.