ON CALENDAR FORMULAS

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For Ralph

ABSTRACT. This paper deals with calendar formulas for calculating the day of the week and the *Julian* day number as used in Astronomy. It describes my implementation of these formulas in a C program as well as in Python.

PREFACE

Ten years ago I published an article on *Gauss*' calendar formula [3], which determines the day of the week of the 1^{st} of January of a year A as

$$w \equiv 5 \cdot (A-1) \bmod 4 + 4 \cdot (A-1) \bmod 100 + 6 \cdot (A-1) \bmod 400 \pmod 7$$

where $w = 0$ is *Monday*, $w = 1$ is *Tuesday*, ..., $w = 6$ is *Sunday* [2, XI, 1.].

Here I treat the *Julian* day number j introduced by *John Herschel* in 1849. The *Gauss* formula can be bypassed by $j(y,m,d)$ as $w \equiv j \pmod 7$.

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1. LEAP YEARS THROUGH THE TIMES

In this article I use the ISO 8601 format for dates: `y-m-d`, in formulas written as (y,m,d). The use of integers for years signifies that I denote 1 BC = 0, etc.

The calendar currently used worldwide is the *Gregorian* calendar, which is derived from the Roman calendar as reformed by *Julius Caesar* in −45 and corrected by *Augustus (Octavian)* in −7. The *Julian* rule prescribes: every four years is a *leap* year. In the early years after the reform by mistake every third year was made a leap year and *Augustus* corrected it (leaving out several leap years). Since year 8 the rule was followed until 1582.

Then a reform by Pope *Gregory XIII* refined this rule by dropping the leap day in years divisible by 100, but not by 400. This adjustment was necessary to keep the *vernal equinox* near the 21^{st} of March. To bring the calendar back into synch the *Gregorian* reform also dropped ten days in October: the last day of the *Julian* calendar was Thursday, 4 October 1582 and was followed by the first day of the *Gregorian* calendar: Friday, 15 October 1582.

The dates up to `1582-10-04` are treated as *Julian* calendar dates - as if in use since eternity (so called *proleptic* calendar). Keep in mind, though, this is historically not correct before *Caesar*'s time; also, the current numbering of the years was invented by

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Dionysius Exiguus in 525 and adopted by the venerable *Bede* in 731 and thereafter only slowly promulgated.

Dates since `1582-10-15` are assumed to follow the *Gregorian* rules forever. The year 1582 was shorter than ordinary years by 10 days, it had only 355 days. Finally the dates `1582-10-5`, `1582-10-6`, ... `1582-10-14` are false, they never existed.

2. Gauss ansatz

In [2, XI, 1.] *Gauss* conceives a numbering of the days by starting at some particular date and linearly extending the numeration to the future. For the 1^{st} of January he writes down a simple formula, reflecting the *Gregorian* leap year rules in a supple way ("geschmeidige Art"). His note is reproduced below as appendix A.4.

We will generalize his approach to arbitrary dates (including the *Julian* era) and extending to the past. We will take a different starting point, which was proposed in 1849 by *John Herschel*: the *Julian* day number, starting at `-4712-1-1`. This is used in *Astronomy*, in particular, where it denotes *noon* of a day.

We consider the group operation of $\mathbf{Z}$ on the set $T \subset \mathbf{Z} \times \{1,\dots,12\} \times \{1,\dots,31\}$ of dates as in [3, §2.3 shifting dates]:

$$\begin{aligned} \mathbf{Z} \times T &\longrightarrow T \\ (n,t) &\longmapsto n+t \end{aligned}$$

where $n+t$ is defined as t shifted by n days, earlier ($n<0$) or later ($n>0$) in time. For a year $a \in \mathbf{Z}$ we let $T_a \subset T$ be the finite set of dates of that year: $t=(y,m,d) \in T_a \Leftrightarrow y=a$. If a is a *leap* year, the cardinal is $|T_a|=366$, otherwise $|T_a|=365$, except for $|T_{1582}|=355$. We can also write $|T_a| = 365+\ell(a)$ for all $a \neq 1582$, where $\ell(a)=1$ for a *leap* year and $\ell(a)=0$ otherwise.

We have obvious maps (*offset*) $x : T_a \longrightarrow \{1,\dots,366\}$ counting the days of a year from the first to the last. When $e(m)$ is the number of days in previous months

m	1	2	3	4	5	6	7	8	9	10	11	12
$e(m)$	0	31	59	90	120	151	181	212	243	273	304	334

then for $t \in T_a$, $a \neq 1582$, $x(t) = \begin{cases} d+e(m) & \text{for } m \leq 2 \\ d+e(m)+\ell(a) & \text{for } m > 2 \end{cases}$

Let $z : T \longrightarrow \mathbf{Z}$ be any *linear* map, such that $z(n+t) = n+z(t)$. There is *essentially* one, only depending on the *origin* t_0 such that $z(t_0)=0$. The choice $t_0=(-4712,1,1)$ will be denoted by j, the *Julian* day number.

For two dates $t,t' \in T$, $t=(a,m,d)$, $t'=(a',m',d')$ and $t<t'$ we have

$$z(t')-z(t) = x(t')-x(t)+\sum_{y=a}^{a'-1}|T_y|$$

and by the above (assuming $y \neq 1582$) we have

$$\sum_{y=a}^{a'-1}|T_y| = (a'-a)\cdot 365 + \sum_{y=a}^{a'-1}\ell(y)$$

The last sum, the number of leap years in the interval $[a,a'-1]$, is written in the *supple way* by *Gauss* for $a \equiv 1 \bmod 400$

$$\sum_{y=a}^{a'-1}\ell(y) = \left[\frac{a'-a}{4}\right] - \left[\frac{a'-a}{100}\right] + \left[\frac{a'-a}{400}\right]$$

which simply reflects the *Gregorian* leap year rule. Hence

$$z(t')-z(t)=x(t')-x(t)+(a'-a)\cdot 365+\left[\frac{a'-a}{4}\right]-\left[\frac{a'-a}{100}\right]+\left[\frac{a'-a}{400}\right]$$

The interesting point now is, that this formula is not only true for $t<t'$ but for $t>t'$ as well, as will be derived in the next section.

3. Counting intervals in congruence classes

We prove some simple properties of the *Gauss floor* function $[x]$, which he introduced in 1808 (see [1], p. 459).

Lemma 3.1. *For $m,n\in\mathbf{Z}$, $m>0$ we have*

$$\left[\frac{-1-n}{m}\right]=-1-\left[\frac{n}{m}\right]$$

Proof. We write $n=q\cdot m+r$ with $0\le r<m$ such that $q=[n/m]$. Then $0\le m-r-1<m$ and as $-1-n=-(q+1)\cdot m+(m-r-1)$ we have $[(-1-n)/m]=-q-1$. □

Lemma 3.2. *For $a,n,m\in\mathbf{Z}$, $n>0$, $m>0$ with $a\equiv 1 \pmod m$ we have*

$$|[a,a+n-1]\cap m\mathbf{Z}|=\left[\frac{n}{m}\right] \tag{1}$$

$$|[a-n,a-1]\cap m\mathbf{Z}|=-\left[\frac{-n}{m}\right] \tag{2}$$

Proof. For (1): let $n=q\cdot m+r$ with $0\le r<m$ and $a=1+k\cdot m$. The set

$$[a,a+n-1]\cap m\mathbf{Z}=\{a-1+m,\dots,a-1+q\cdot m=a+n-1-r\}$$

has $q=[n/m]$ elements.

For (2): let $n-1=q\cdot m+r$ with $0\le r<m$ and $a=1+k\cdot m$. Then $a-n=k\cdot m-q\cdot m-r=(k-q)\cdot m-r$ and the set

$$[a-n,a-1]\cap m\mathbf{Z}=\{a-n+r,a-n+r+m,\dots,a-n+r+q\cdot m=a-1\}$$

contains $1+q$ elements, which by lemma 3.1 is $=1+[(n-1)/m]=-[-n/m]$. □

We will apply this to the *leap* years in the *Julian* resp. *Gregorian* era in the interval $a\le y<a'$ for $a<a'$ and in the interval $a'\le y<a$ for $a'<a$.

In the *Julian* case this set is $[a,a'-1]\cap 4\mathbf{Z}$ for $a<a'$ and $[a',a-1]\cap 4\mathbf{Z}$ for $a'<a$.

In the *Gregorian* case this set is $[a,a'-1]\cap 4\mathbf{Z}\setminus[a,a'-1]\cap 100\mathbf{Z}\cup[a,a'-1]\cap 400\mathbf{Z}$ for $a<a'$ and $[a',a-1]\cap 4\mathbf{Z}\setminus[a',a-1]\cap 100\mathbf{Z}\cup[a',a-1]\cap 400\mathbf{Z}$ for $a'<a$. Remark that the union is *disjoint*.

Corollary 3.3. *Let $t,t'\in T$, $t=(a,m,d)$, $t'=(a',m',d')$.*

In the Julian *era: $t,t'\le$ `1582-10-04`, for $a\equiv 1 \pmod 4$*

$$z(t')-z(t)=x(t')-x(t)+(a'-a)\cdot 365+\left[\frac{a'-a}{4}\right] \tag{3}$$

In the Gregorian *era: $t,t'\ge$ `1582-10-15`, for $a\equiv 1 \pmod{400}$*

$$z(t')-z(t)=x(t')-x(t)+(a'-a)\cdot 365+\left[\frac{a'-a}{4}\right]-\left[\frac{a'-a}{100}\right]+\left[\frac{a'-a}{400}\right] \tag{4}$$

Proof. The corollary is clear for $a < a'$. For $a' < a$ we remark that $z(t) - z(t') = x(t) - x(t') + (a-a')\cdot 365 + \sum_{y=a'}^{a-1} \ell(y)$ and applying Lemma 3.2 (2) with $n = a - a'$ we get

$$z(t) - z(t') = x(t) - x(t') + (a-a')\cdot 365 - \left[\frac{a'-a}{4}\right] + \left[\frac{a'-a}{100}\right] - \left[\frac{a'-a}{400}\right]$$

with the last two terms only in the *Gregorian* case. Multiplying by -1 gives the asserted formula. □

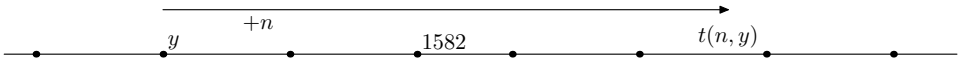
4. Programming notes

In this section I explain some points of the program `cal.c` in appendix A.2.

There are four functions: `main`, `date`, `leap` and `quot`, the latter three reused from `gauss` [3]. `leap` implements the *leap* function $\ell(y)$ and `quot` the *floor* function $[n/m]$ for $m > 0$. Function `date` needs some explanation.

4.1. **Function `date`.** It is called with an *offset* n and by setting y to a *starting* year. Think of it as a *linear* function $t : \mathbf{Z}\times\mathbf{Z} \to T$, $t(n,y) = (a,m,d)$ with $t(k+n,y) = k + t(n,y)$. It is understood that $n = 1$ corresponds to the first day of the year y, $t(1,y) = (y,1,1)$, hence offset $n = 0$ corresponds to New Year's Eve, $t(0,y) = (y-1,12,31)$.

Another special point is that I have *not* implemented the exceptional year 1582, which only has 355 days. That is, I have to avoid situations like the following, crossing the critical year. In the figure I start in the *Julian* era and add an offset getting me to the *Gregorian* era.



Similarly, situations have to be avoided starting in the *Gregorian* era and taking us back to the *Julian* era by a *negative* offset.

4.2. **Some *Julian* day numbers.** We need to understand some of the constants in the `main` function, gathered in this table

$$\begin{array}{lll} j(0,12,31) = 1,721,423 & j(1581,12,31) = 2,298,883 & j(1582,10,4) = 2,299,160 \\ j(1582,10,15) = 2,299,161 & j(1582,12,31) = 2,299,238 & j(2000,12,31) = 2,451,910 \end{array}$$

Using (3) with $t' = (-4712,1,1)$ and $t = (1,1,1)$ we get

$$j(-4712,1,1) - j(1,1,1) = -4713\cdot 365 + \left[\frac{-4713}{4}\right] = -1720245 - 1179 = -1721424$$

As by definition $j(-4712,1,1) = 0$ we obtain $j(1,1,1) = 1721424$.

The next value is $j(1582,1,1) = j(1,1,1) + 1581\cdot 365 + [1581/4] = 1721424 + 577065 + 395 = 2298884$. $j(1582,10,4) = j(1581,12,31) + 4 + e(10) = 2298883 + 4 + 273 = 2299160$. $j(1582,12,31) = j(1581,12,31) + 355 = 2299238$. Using (4) we have $j(2001,1,1) = j(1583,1,1) + 418\cdot 365 + 1 + [418/4] - [418/100] + [418/400] = 2299239 + 152570 + 1 + 104 - 4 + 1 = 2451911$.

4.3. **The `main` function.** The first half of the `main` function does housekeeping and parameter checking.

The following lines

```
if (y < 1582 || y == 1582 && m*100+d < 1005) // Julian era
   j = 1721423 + d + e[m] + (y-1)*365 + quot(y-1,4);
else {                                  // Gregorian era
   x = y - 2001;
   j = 2451910 + d + e[m] + x*365 + quot(x,4) - quot(x,100) + quot(x,400);
}
```

set the *Julian* day number $j = j(t)$ depending on the era, making use of the formulas (3) resp. (4), with the values just calculated.

The next two lines adjust for leap year and add the *offset* from the command line.

```
   if (m>2) j+=leap(y);                // correction for leap year
   j += n;                             // add offset
```

Eventually, the correct dates are calculated by calling function `date` either for a *Julian* date starting with 1582, or for a *Gregorian* date starting with year 1583 – to avoid the crossing of the critical year described above.

```
if (j < 2299161) {                   // Julian era
   y=1582;
   date(j-2298883,&y,&m,&d);         // calculate the date
}
else {                               // Gregorian era
   y=1583;
   date(j-2299238,&y,&m,&d);         // calculate the date
}
```

The remainder of the `main` function is obvious.

APPENDIX A.

A.1. **The program `cal`.** The input syntax is `date [offset]`, where *date* is in ISO 8601 format `year-mm-dd` and *offset* can be any integer. Weeks run from Monday to Sunday, its numbering follows ISO.

The program checks for correct dates under the assumption that

- all dates previous to `1582-10-04` are dates in the proleptic *Julian* calendar
- all dates past `1582-10-15` are dates in the proleptic *Gregorian* calendar
- the dates in the range `1582-10-05` to `1582-10-14` are rejected as incorrect

The program `cal` displays

- the day of the week (Mon, Tue, Wed, Thu, Fri, Sat, Sun)
- the date (*Julian* for ≤ `1582-10-04`, *Gregorian* for ≥ `1582-10-15`)
- the Julian day number (`J#`)
- the day of the year (`D#`)
- the week of the year (`W#`)

The C program operates correctly in the range of dates `-5,877,908-3-15` to `+5,874,898-6-3` corresponding to values of j in the range $-2^{31}-1+2,298,883 \le j \le 2^{31}-1 = 2,147,483,647$. This range originates from the overflow in the type `long int` of j.

A.2. **Program listing `cal.c`.**

```
/* ------------------------------------------------------------------

   Module:       cal.c

   Description:  Calendar formulas

        Input:   year-mm-dd [offset] (ISO 8601 Format)

        Output:  day of the week, date, Julian day number,
```

```
                day of the year, week of the year

   Example: cal 1777-04-30 +84005
            Mon 2007-04-30 J# 2454221 D# 120 W# 18

   Documentation:
        http://berndt-schwerdtfeger.de/wp-content/uploads/pdf/calj.pdf

   ------------------------------------------------------------------
```



```
   ------------------------------------------------------------------*/

#include <stdio.h>
#include <stdlib.h>

// set constants

char *wd[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};
int  e[13] = {0,0,31,59,90,120,151,181,212,243,273,304,334};

// function prototypes

void date(long,long*,int*,int*);      // offset, year, month, day
int  leap(long);                      // = 1 if year is a leap year
long quot(long,long);                 // = [a/b] (corrected: works if a < 0)

// ------------------------------------------------------------------
// main function
// ------------------------------------------------------------------

int main(int argc, char *argv[]){

   if (argc == 1 || *argv[1] == '?'){
      printf("\ncal v1.3.2, 2023-12-25 \n");
      printf("-- Copyright (C) 2010-2023 Berndt E. Schwerdtfeger -- \n\n");
      printf("Input:  year-mm-dd [offset] \n\n");
      printf("Output: {Mon|..|Sun} year-mm-dd J# jjjjjjj D# nnn W# ww\n\n");
      printf("Example: cal 1777-04-30 +84005 \n");
```

```
      printf("Mon␣2007-04-30␣J#␣2454221␣D#␣120␣W#␣18\n\n");
      return EXIT_SUCCESS;                  // end the program
   }                                        // argc > 1 here
   int  w,m,d;
   long x,y,j,n=0;

   sscanf(argv[1],"%ld-%d-%d",&y,&m,&d);

   if (argc == 3)
      sscanf(argv[2],"%ld",&n);             // get the offset
   else if (argc > 3) {
      printf("Too␣many␣parameters␣!\n");
      return EXIT_FAILURE;                  // end the program
   }
   if (y==1582 && m==10 && d>4 && d<15) {
      printf("This␣date␣is␣invalid,␣it␣did␣never␣exist␣!\n");
      return EXIT_FAILURE;                  // end the program
   }
   if (y < 1582 || y == 1582 && m*100+d < 1005) // Julian era
      j = 1721423 + d + e[m] + (y-1)*365 + quot(y-1,4);
   else {                                   // Gregorian era
      x = y - 2001;
      j = 2451910 + d + e[m] + x*365 + quot(x,4) - quot(x,100) + quot(x,400);
   }
   if (m>2) j+=leap(y);                     // correction for leap year
   j += n;                                  // add offset
   if (j < 2299161) {                       // Julian era
      y=1582;
      date(j-2298883,&y,&m,&d);             // calculate the date
   }
   else {                                   // Gregorian era
      y=1583;
      date(j-2299238,&y,&m,&d);             // calculate the date
   }
   n = d + e[m];                            // offset into this year
   if (m > 2)
      n += leap(y);                         // adjust for leap day
   if (y==1582 && n>278) n-=10;             // ten days dropped in 1582
   w = j%7;                                 // set weekday
   if (w<0) w+=7;
   printf("%s␣%ld-%02d-%02d␣J#␣%ld␣", wd[w],y,m,d,j);
   printf("D#␣%03d␣W#␣%02d\n", n,(n-w+9)/7);
   return EXIT_SUCCESS;                     // exit the program
};

// ---------------------------------------------------------------------
// function definitions
// ---------------------------------------------------------------------

void date(long n, long* y, int* m, int* d){

   int i;
```

```
   while (n > 365 + leap(*y)){          // if offset larger than # of
      n -= 365 + leap(*y);              // ... days in a year
      *y += 1;                          // ... find the correct year
   }
   while (n <= 0){                      // if offset is negative
      *y -= 1;                          // ... find the correct year
      n += 365 + leap(*y);              // ... and offset
   }
   i = leap(*y);                        // adjust for leap day
   *m = 12;
   while (n <= e[*m] + i){              // searching for the month
      *m -= 1;
      if (*m<3) i=0;
   }
   *d = n - e[*m] - i;                  // setting the day
}

// -----------------------------------------------------------------------
int leap(long y){

   int i = 0;
   if (y%4 == 0)
      i = 1;
   if (y > 1582 && y%100 == 0 && y%400 != 0)
      i = 0;
   return i;
}

// -----------------------------------------------------------------------
long quot(long a, long b){

   long x = a/b;
   if (a < 0 )                          // for negative numerator ..
      x -= (a%b != 0);                  // .. if remainder, subtract 1
   return x;
}
```

A.3. **Program listing** `cal.py`**.** The same remarks as for the C program above apply.

```
#!/usr/bin/env python3
# -----------------------------------------------------------------------
#
#   Module:       cal.py
#
#   Description: Calendar formulas
#
#         Input:  year-mm-dd [offset]
#
#         Output: day of the week, date, Julian day number,
#                 day of the year, week of the year
#
#   Example: cal 1777-04-30 +84005
#            Mon 2007-04-30 J# 2454221 D# 120 W# 18
```

```
#
#   Documentation:
#       http://berndt-schwerdtfeger.de/wp-content/uploads/pdf/calj.pdf
#
# ------------------------------------------------------------------------
#   Copyright (C) 2023 Berndt E. Schwerdtfeger
#
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#   WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied.
#   See the License for the specific language governing permissions and
#   limitations under the License.
# ------------------------------------------------------------------------

# ------------------------------------------------------------------------
# function definitions
# ------------------------------------------------------------------------

def leap(y):
    leap = False
    if   y%4   == 0 :
        leap = True
    if (y > 1582 and y%100 == 0 and y%400 != 0):
        leap = False
    if leap:
        return 1
    else:
        return 0

# ------------------------------------------------------------------------
def date(n):
    global y,m,d
    while (n > 365 + leap(y)):          # if offset larger than # of
        n -= 365 + leap(y)              # ... days in a year
        y += 1                          # ... find the correct year

    while (n <= 0):                     # if offset is negative
        y -= 1                          # ... find the correct year
        n += 365 + leap(y)              # ... and offset
    i = leap(y)                         # adjust for leap day
    m = 12
    while (n <= e[m] + i):              # searching for the month
        m -= 1
        if (m<3): i=0
    d = n - e[m] - i;                   # setting the day


# ------------------------------------------------------------------------
```

```
# main program
# ------------------------------------------------------------------
import sys

argc=len(sys.argv)
if (argc==1 or sys.argv[1]  == '?'):
    print("\ncal v1.3.2, 2023-12-25")
    print("-- Copyright (C) 2010-2023 Berndt E. Schwerdtfeger --\n")
    print("Input:  year-mm-dd [offset]");
    print("Output: {Mon|..|Sun} year-mm-dd J# jjjjjjj D# nnn W# ww");
else:
    s=sys.argv[1]
    # set constants  -- move them to some spot below
    wd = ["Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"]
    e = [0,0,31,59,90,120,151,181,212,243,273,304,334]

    if (argc >= 3):
        n=int(sys.argv[2])
    else:                                   # if no offset provided
        n=0                                 # ... initialize to 0

    # parse the argument, should be in format y-m-d, with
    # y integer (so: may have a minus sign in front!)
        # m in the range 1 <= m <= 12
        # d in the range 1 <= d <= 31

    ymd = s.split('-')
    if (len(ymd[0])==0):                    # vorzeichen
        y = -int(ymd[1])
        m = int(ymd[2])
        d = int(ymd[3])
    else:
        y = int(ymd[0])
        m = int(ymd[1])
        d = int(ymd[2])

    if (y==1582 and m==10 and d>4 and d<15):
        print("Invalid date")
        sys.exit()                          # abort
    # test if we are in Julian or Gregorian era
    if (y < 1582 or y == 1582 and m*100+d < 1005):
        j = 1721423 + d + e[m] + (y-1)*365 + (y-1)//4
    else:                                   # Gregorian era
        x = y - 2001
        j = 2451910 + d + e[m] + x*365 + x//4 - x//100 + x//400

    if (m>2): j+=leap(y)                    # correction for leap year
    j += n                                  # add offset
    if (j < 2299161):                       # Julian era
        y=1582
        date(j-2298883)                     # calculate the date
    else:                                   # Gregorian era
```

```
        y=1583
        date(j-2299238)                 # calculate the date
    n = d + e[m]                        # offset into this year
    if (m > 2): n += leap(y)            # adjust for leap day
    if (y==1582 and n>278): n-=10       # ten days dropped in 1582
    w = j%7                             # set weekday
#   if (w<0): w+=7                        does not occur in Python
    print('%s %d-%d-%d J# %d D# %d W# %d' % (wd[w],y,m,d,j,n,(n-w+9)/7))
```

A.4. **Gauss' calendar formula in his Nachlass.** *Gauss* did not publish his formula for finding the day of the week. It appeared posthumously in his *Werke* in 1927 [2, XI, p. 206]. The *bracket* notation for the floor function $[x]$ was not yet used, *Gauss* introduced it in 1808.

Den Wochentag des 1. Januar eines Jahres zu finden.

Handschriftliche Eintragung in: Sammlung astronomischer Tafeln, unter Aufsicht der Kgl. Preußischen Akademie der Wissenschaften, I. Band, Berlin 1776. — 1798 von GAUSS erworben.

Bezeichnet man den kleinsten positiven Rest einer Grösse A nach dem Modulus m durch $R:A \pmod m$, so lassen sich alle Vorschriften des Gregorianischen Kalenders auf folgende geschmeidige Art darstellen:

1.

Wenn man die Tage vom 1^{ten} Januar 1701 an zählt, d. i. diesen mit 1, den 2^{ten} mit 2, den 31^{ten} Dec. 1700 mit 0, den 30^{ten} mit -1 etc. bezeichnet, so ist der 1^{te} Januar in irgend einem Jahre A

$$\begin{aligned} = 1 + (A-1701)365 &+ \frac{1}{4}\big((A-1701) - R:(A-1701) \bmod 4\big) \\ &- \frac{1}{100}\big((A-1701) - R:(A-1701) \bmod 100\big) \\ &+ \frac{1}{400}\big((A-1601) - R:(A-1601) \bmod 400\big) \end{aligned}$$

2.

Die Wochentage Sonntag, Montag, etc. mit 0, 1 etc. bezeichnet, ist der 1^{te} Januar irgend eines Jahres, qua Wochentag,

$$\left.\begin{aligned} \equiv 6 + A &+ (2A+5R:(A-1) \bmod 4) \\ &+ (3A+4R:(A-1) \bmod 100) \\ &+ (A+2+6R:(A-1) \bmod 400) \end{aligned}\right\} \bmod 7.$$

Also von 1601 bis 2000

$$\equiv 6 + 6A + 5R:(A-1) \bmod 4 + 4R:(A-1) \bmod 100.$$

Allgemein

$$\equiv 1 + 5R:(A-1) \bmod 4 + 4R:(A-1) \bmod 100 + 6R:(A-1) \bmod 400.$$

REFERENCES

[1] Carl Friedrich Gauss, *Untersuchungen über höhere Arithmetik*, 2nd ed., AMS Chelsea Publishing, Providence, 1889. reprinted 1981.

[2] ———, *Werke* (1863), available at http://gdz.sub.uni-goettingen.de/id/PPN235957348.

[3] Berndt E. Schwerdtfeger, *Gauss' calendar formula for the day of the week* (1999), available at http://berndt-schwerdtfeger.de/wp-content/uploads/pdf/cal.pdf.